APPLICAZIONI OLOMORFE TRA SPAZI SIMMETRICI

Parma, 28 Aprile 2005

ESEMPI

Esempio 1. $(\mathbb{C}^n, \omega = \frac{i}{2}\sum_{j=1}^n dz_j \wedge d\bar{z}_j)$

$$\omega = \frac{i}{2}\partial\bar{\partial}|z|^2 \Longrightarrow \Phi(z) = |z|^2 \Longrightarrow \tilde{\Phi}(z, \bar{w}) = z\cdot\bar{w}$$

$$D(z, w) = |z|^2 + |w|^2 - z\cdot\bar{w} - w\cdot\bar{z} = |z-w|^2$$

Esempio 2. $(\mathbb{C}P^n, \omega_{FS}),\ n \leq \infty$

$$\omega_{FS} = \frac{i}{2}\partial\bar{\partial}|z|^2,\ |z|^2 = |z_0|^2 + \cdots + |z_n|^2$$

$$\Phi([z]) = |z|^2 \Longrightarrow \tilde{\Phi}([z], \overline{[w]}) = z\cdot\bar{w}$$

$$D([z], [w]) = \log\frac{|z|^2|w|^2}{|z\cdot\bar{w}|^2}$$

Esempio 3. $(\mathbb{C}H^n, \omega_{hyp}),\ n \leq \infty$

$$\mathbb{C}H^n = \{z \in \mathbb{C}^n |\ |z|^2 < 1\}$$

$$D(0, z) = \log(1 - |z|^2)$$

RISULTATI SUGLI SPAZI SIMMETRICI

Teorema 1. (Calabi, Ann. of Math. 1953)

$$\nexists\ \mathbb{C}H^k_b \to \mathbb{C}^n$$

$$\nexists\ \mathbb{C}^k \to \mathbb{C}P^n_b$$

$$\nexists\ \mathbb{C}H^k_b \to \mathbb{C}P^n_b$$

Teorema 2. (Calabi, Ann. of Math. 1953)

$$F(n,b) \xrightarrow{f} F(n',b') \quad \Longrightarrow \quad F(n,b) \overset{f}{\hookrightarrow} F(n',b')$$

Teorema 3. (H. Nakagawa and R. Takagi, J. Math. Soc. Japan 28, 1976)

LocS $\xrightarrow{f} \mathbb{C}^n$ (oppure LocS $\xrightarrow{f} \mathbb{C}H^n$) $\Longrightarrow f$ tot. geo

Teorema 4. (M. Takeuchi, Japan J. Math 4, 1978)

LocS completo $\xrightarrow{f} \mathbb{C}P^n \Longrightarrow$ LocS $= S_+ \overset{f}{\hookrightarrow} \mathbb{C}P^n$

Definizione.(A. Loi, Diff. Geom. Appl. 2005)

(V,ω) *almost projective-like* $\Longleftrightarrow e^{-D(p,\cdot)} : V \to \mathbb{R}$

(V,ω) *projective-like* $\Longrightarrow$ a.p-l $e^{-D(p,q)} = 1 \Longleftrightarrow p = q$

Proposizione 1. Sia (V,ω) tale che il suo rivestimento universale $(\tilde{V},\tilde{\omega})$ sia projective-like e sia (W,Ω) almost projective-like.

$$(V,\omega) \xrightarrow{f} (W,\Omega) \implies (V,\omega) = (\tilde{V},\tilde{\omega}) \overset{f}{\hookrightarrow} (W,\Omega)$$

Teorema 5 S e $F(n,b)$ sono projective-like.

Corollario 1.

$$\text{LocS completo} \xrightarrow{f} \widehat{S} \implies \text{LocS} = S \overset{f}{\hookrightarrow} \widehat{S}$$

Corollario 2.

$$\text{LocS completo} \xrightarrow{f} F(n,b) \implies \text{LocS} = S \overset{f}{\hookrightarrow} F(n,b)$$

Teorema 6. (A. Loi, Diff. Geom. Appl. 2005)

$$\nexists\ S_i \to S_j,\ i \neq j,\ i, j = -,\ 0,\ +$$

Osservazioni

$$\text{Cor. 1 e Cor. 2} \Longrightarrow \text{Teor. } 2, 3, 4$$

$$\text{Teor. 6} \Longrightarrow \text{Teor. 1}$$